Lesson 9

Approximate Methods for Analysis of Building Frames

Instructional Objectives:

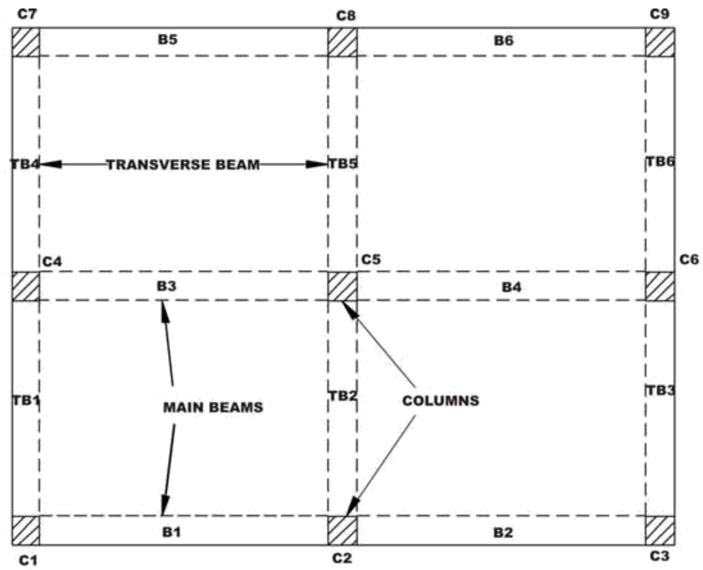
After reading this chapter the student will be able to

1. Analyse building frames by approximate methods for vertical loads.
2. Analyse building frames by the cantilever method for horizontal loads.
3. Analyse building frame by the portal method for horizontal loads.

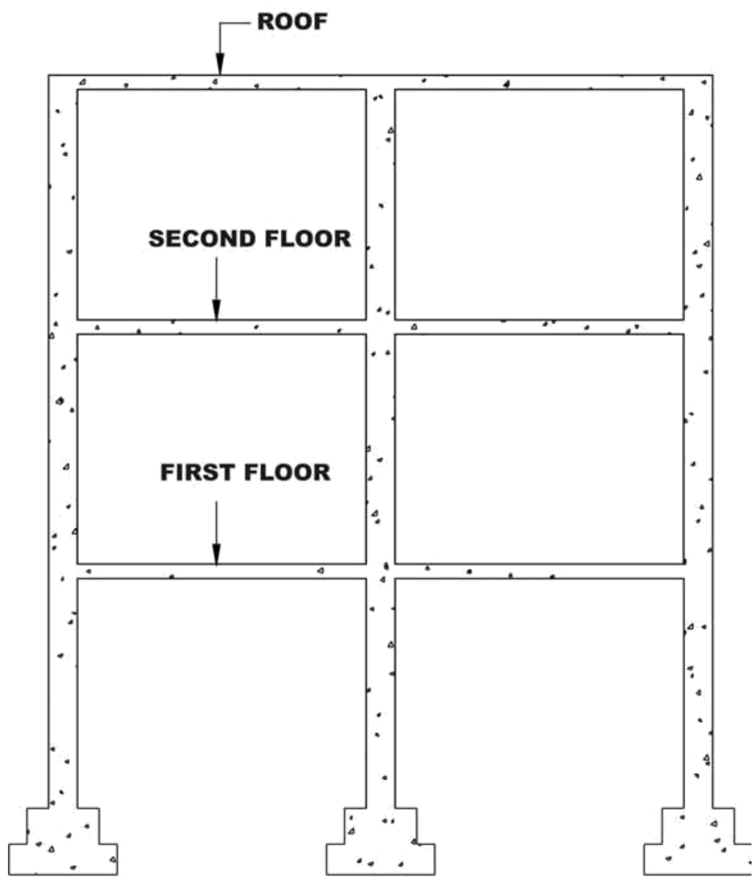
Introduction

Building frames are the most common structural form, an analyst/engineer encounters in practice. Usually the building frames are designed such that the beam column joints are rigid. A typical example of building frame is the reinforced concrete multi-storey frames. A two-bay, three-storey building plan and sectional elevation are shown in Fig. 9.1. In principle this is a three dimensional frame. However, analysis may be carried out by considering planar frame in two perpendicular directions separately for both vertical and horizontal loads as shown in Figure 9.2 and finally superimposing moments appropriately. In the case of building frames, the beam column joints are monolithic and can resist bending moment, shear force and axial force. The frame has 12 joints (j), 15 beam members (b), and 9 reaction components (r). Thus this frame is statically indeterminate to degree $= ((3 \times 15 + 9) - 12 \times 3) = 18$.

Any exact method, such as slope-deflection method, moment distribution method or direct stiffness method may be used to analyse this rigid frame. However, in order to estimate the preliminary size of different members, approximate methods are used to obtain approximate design values of moments, shear and axial forces in various members. Before applying approximate methods, it is necessary to reduce the given indeterminate structure to a determinate structure by suitable assumptions. These will be discussed in this lesson. In section 9.2, analysis of building frames to vertical loads is discussed and in section 9.3, analysis of building frame to horizontal loads is discussed.



Plan



Sectional Elevation Along C_1 - C_3

Figure 9.1 Building Frame

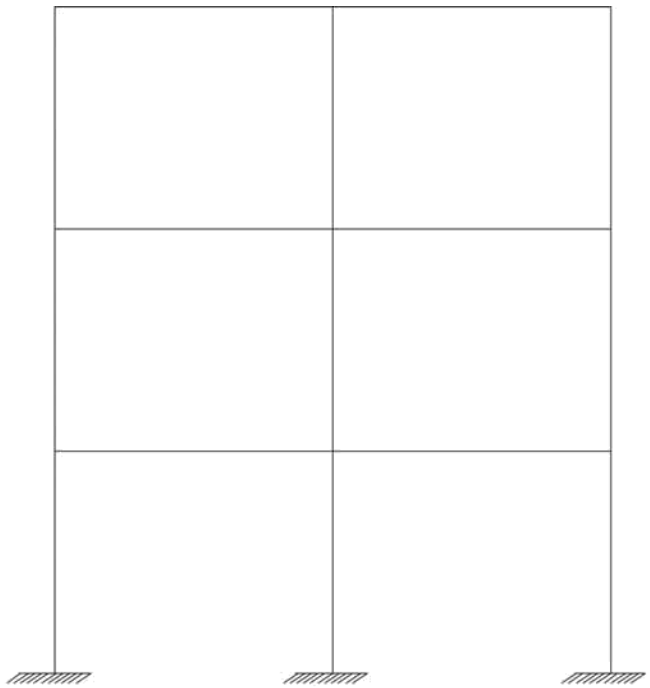


Figure 9.2 Idealized Frame for Analysis

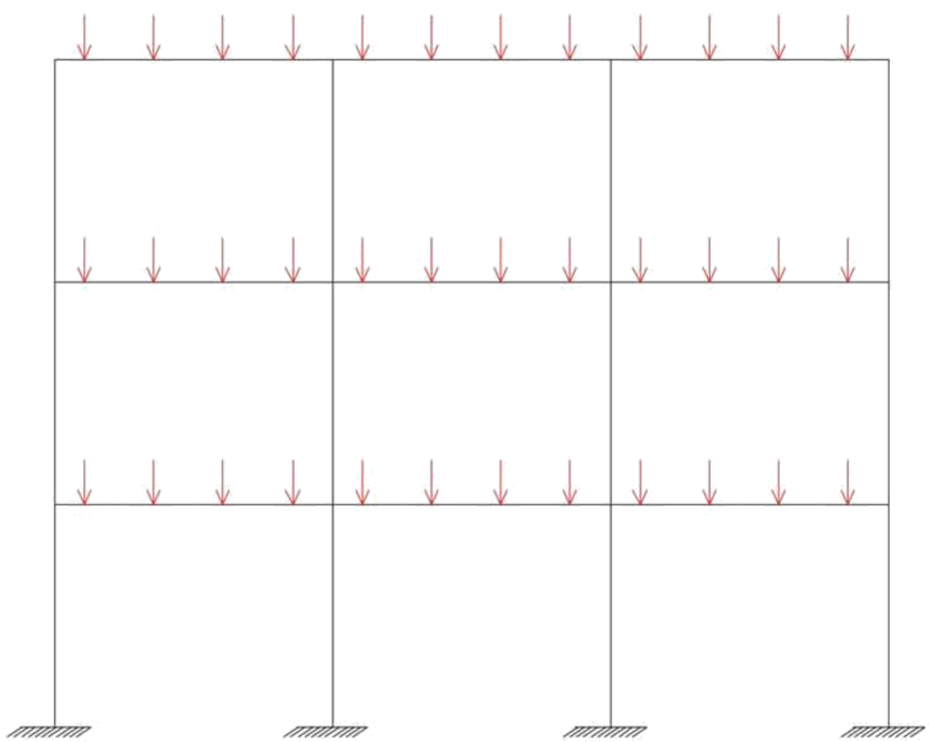


Figure 9.3 Building Frame Subjected to Vertical loads

Analysis of Building Frames subjected to Vertical Loads

Consider a building frame subjected to vertical loads as shown in Figure 9.3. Any typical beam, in this building frame is subjected to axial force, bending moment and shear force. Hence each beam is statically indeterminate to third degree and hence 3 assumptions are required to reduce this beam to a determinate beam.

Before we discuss the required three assumptions consider a simply supported beam. In this case zero moment section (or point of inflexion) occurs at the supports as shown in Figure 9.4a. Next consider a fixed-fixed beam, subjected to vertical loads as shown in Fig. 9.4b. In this case, the point of inflexion or point of zero moment occurs at 0.21L from both ends of the support.

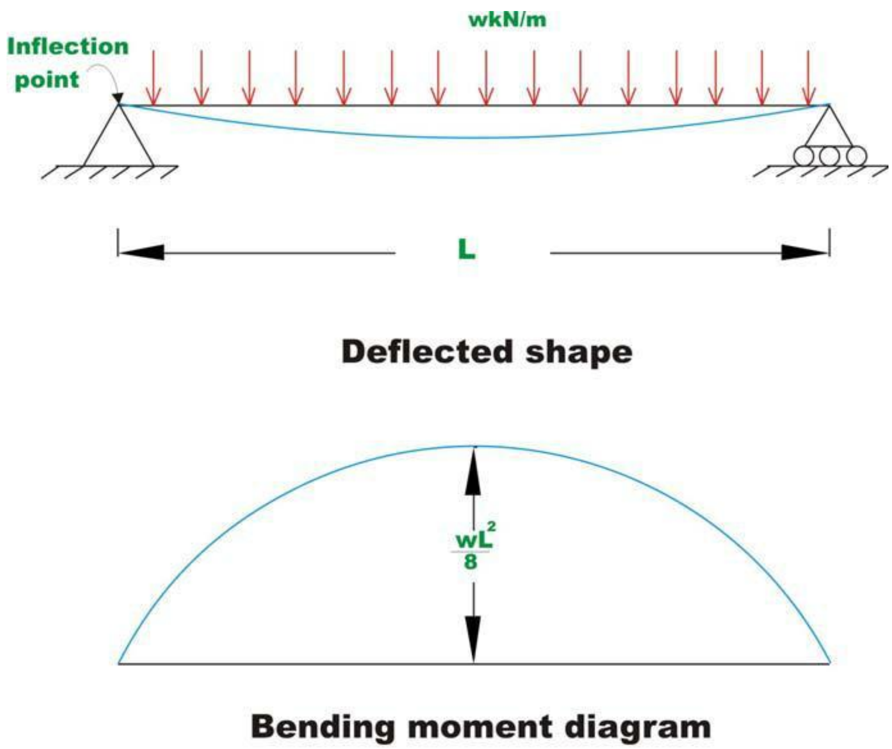
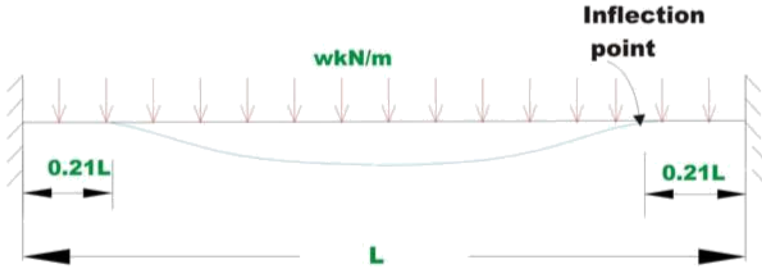
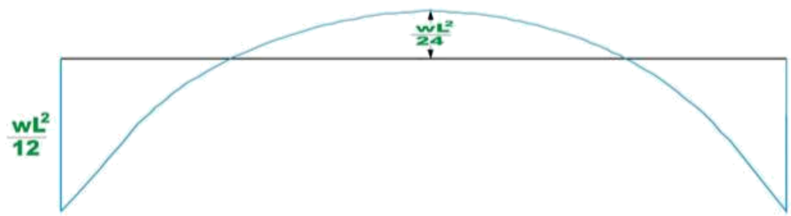


Figure 9.4a Simply Supported beam



Deflected shape



Bending moment diagram

Figure 9.4b Fixed-Fixed beam

Now consider a typical beam of a building frame as shown in Figure 9.4c. In this case, the support provided by the columns is neither fixed nor simply supported. For the purpose of approximate analysis the inflexion point or point of zero moment is assumed to occur at $(0 + 0.21L/2 \approx 0.1L)$ from the supports. In reality the point of zero moment varies depending on the actual rigidity provided by the columns.Thus the beam is approximated for the analysis as shown in Figure 9.4d.

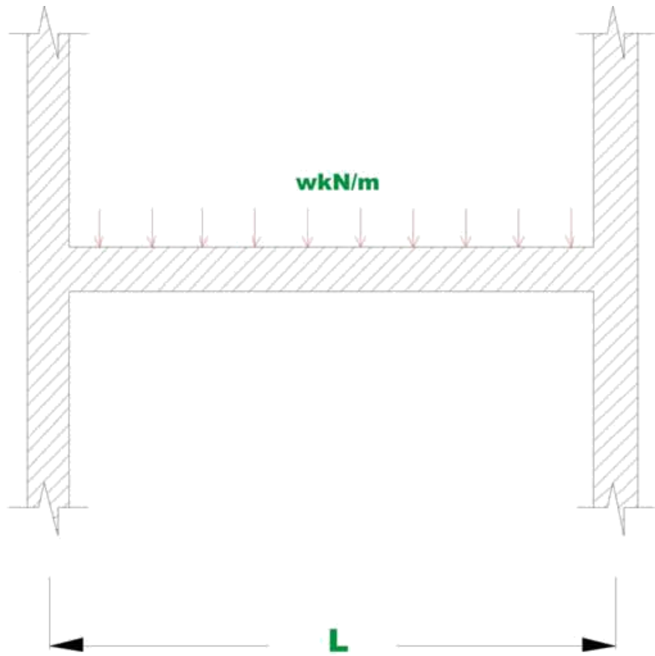


Figure 9.4c

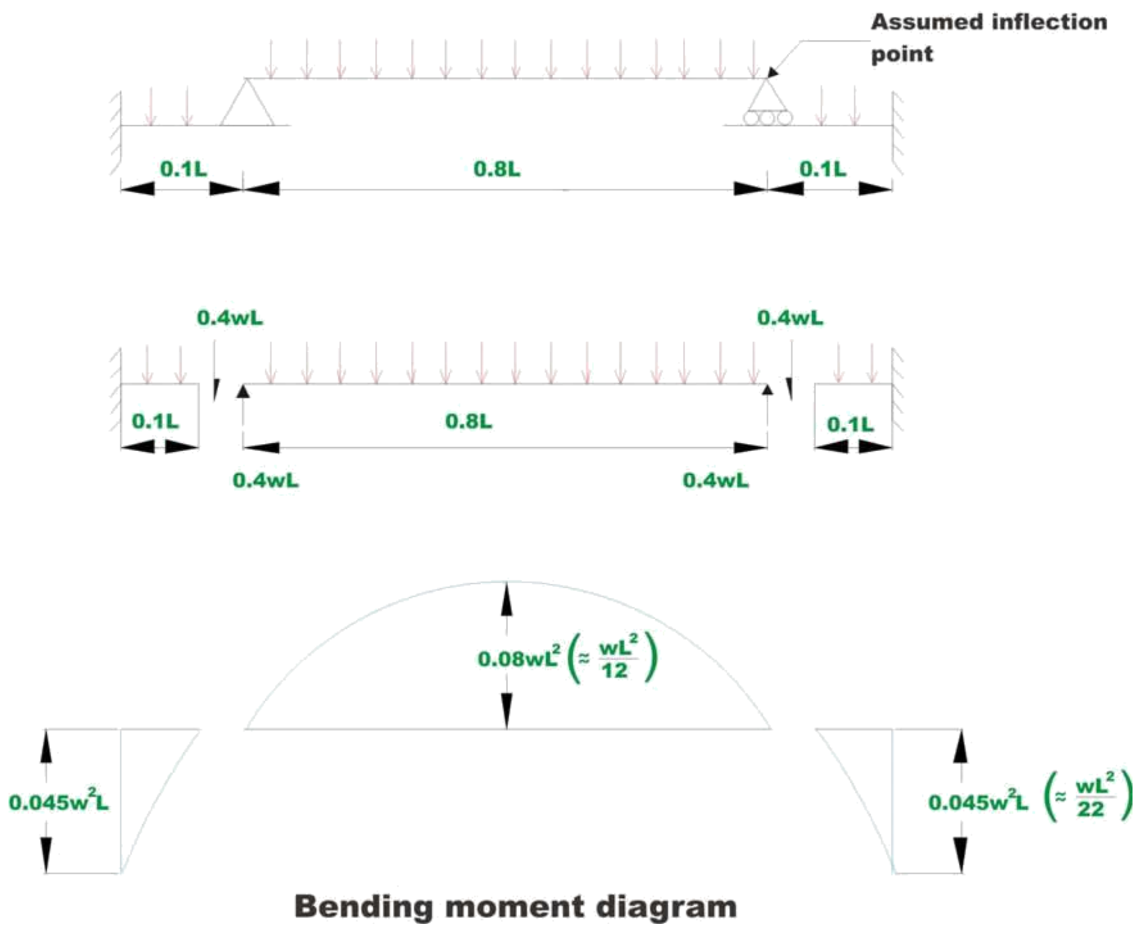


Figure 9.4d

For interior beams, the point of inflexion will be slightly more than $0.1L$. An experienced engineer will use his past experience to place the points of inflexion appropriately. Now redundancy has reduced by two for each beam. The third assumption is that axial force in the beams is zero. With these three assumptions one could analyse this frame for vertical loads.

Example 9.1

Analyse the building frame shown in Figure 9.5a for vertical loads using approximate methods.

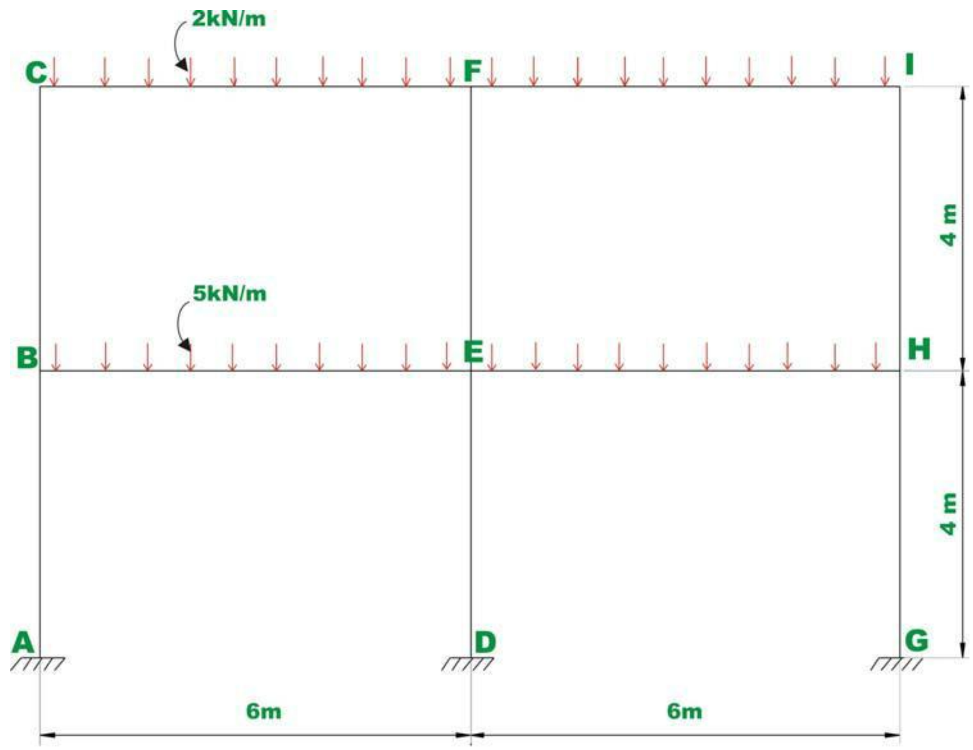


Figure 9.5a

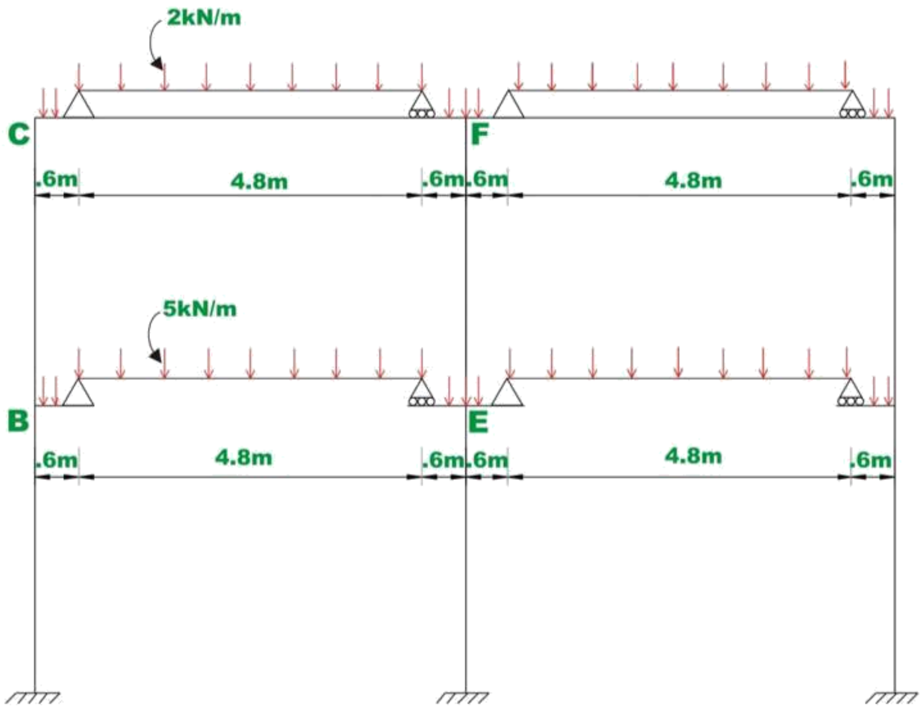


Figure 9.5b

Solution:

In this case the inflexion points are assumed to occur in the beam at $0.1L(= 0.6m)$ from columns as shown in Figure 9.5b. The calculation of beam moments is shown in Figure. 9.5c.

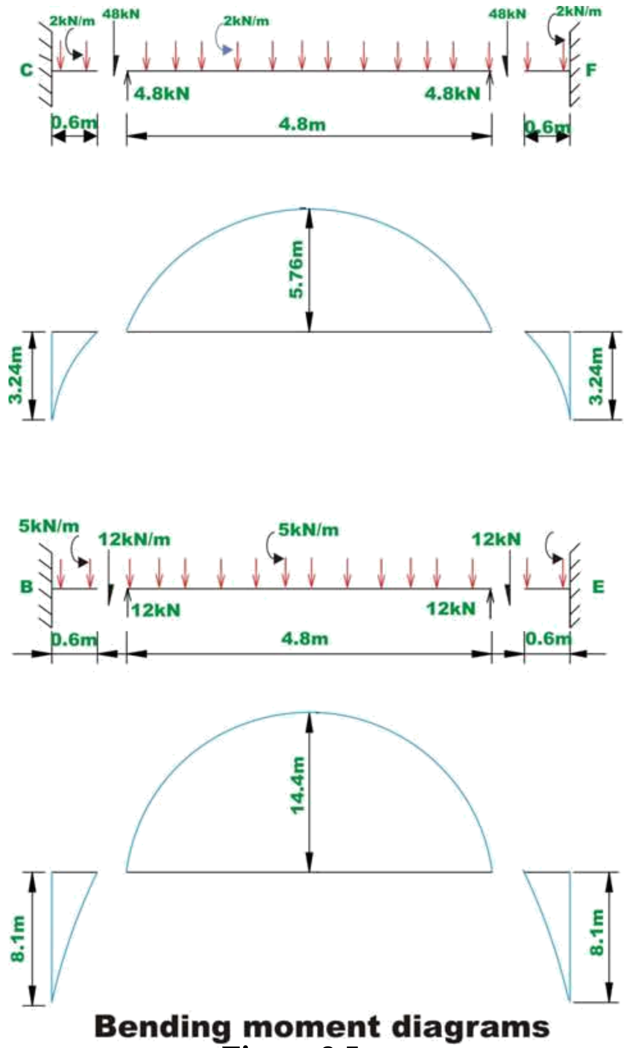


Figure 9.5c

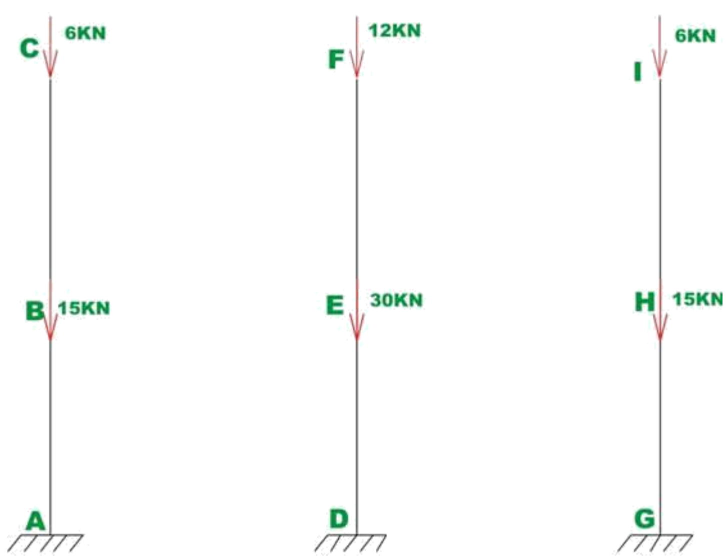


Figure 9.5d Axial Force in columns

Now the beam − *ve* moment is divided equally between lower column and upper column. It is observed that the middle column is not subjected to any moment, as the moment from the right and the moment from the left column balance each other. The (-) ve moment in beam BE is 8.1 kNm. Hence this moment is divided between columns BC and BA. Hence $M_{BC} = M_{BA} = 8.1/2 = 4.05$ kN.m. The maximum (+) *ve* moment in beam BE is 14.4 kN.m . The columns do carry axial loads. The axial compressive loads in the columns can be easily computed. This is shown in Figure 9.5d.

Analysis of Building Frames subjected to Lateral loads

A building frame may be subjected to wind and earthquake loads during its life time. Thus, the building frames must be designed to withstand lateral loads. A two-storey two-bay multi-storey frame subjected to lateral loads is shown in Figure 9.6. The actual deflected shape (as obtained by exact methods) of the frame is also shown in the figure by dotted lines. The given frame is statically indeterminate to degree 12.

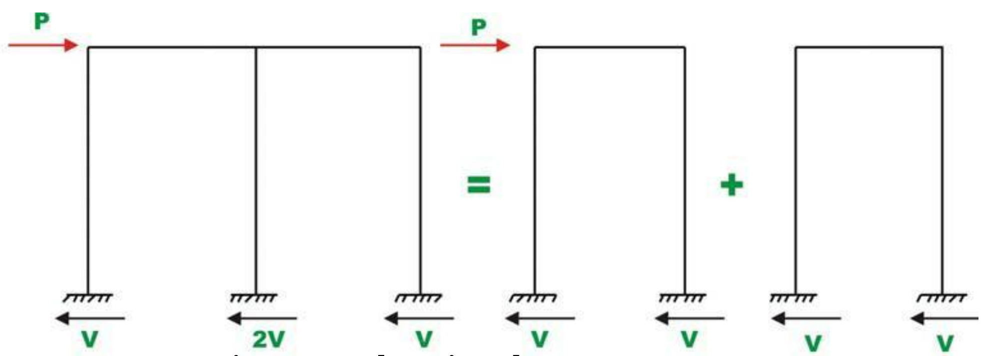


Figure 9.6 Shear in columns

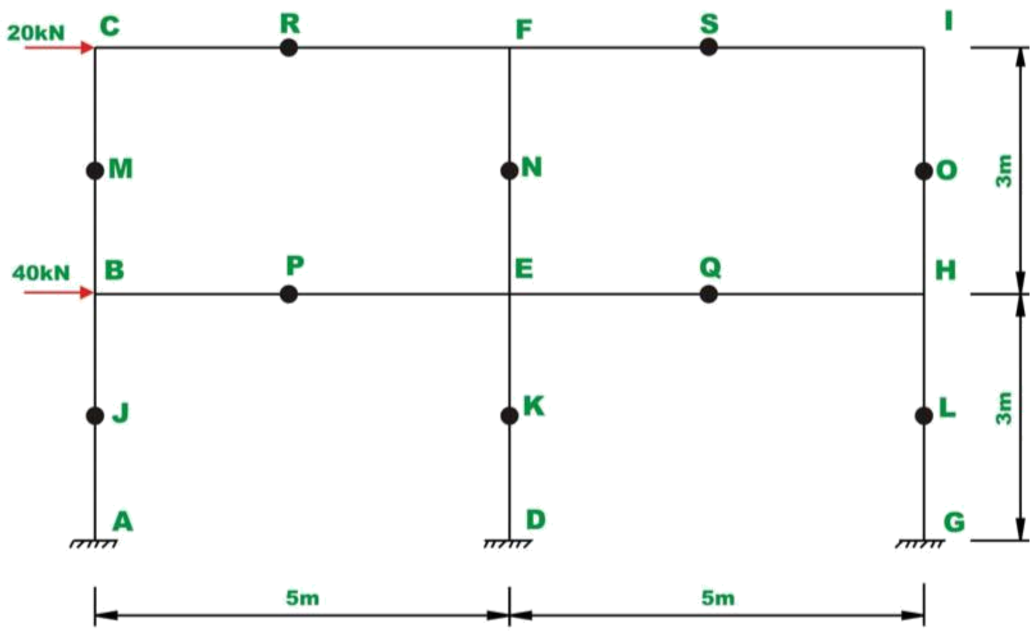


Figure 9.7a Two storey building frame subjected to lateral load of Example 9.2

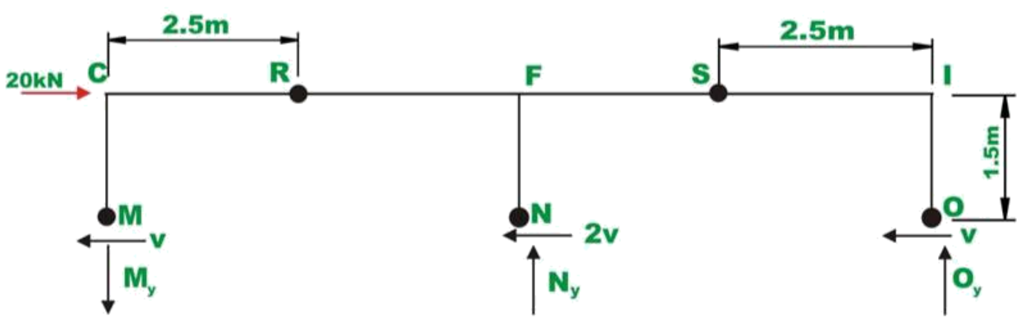


Figure 9.7b

Hence it is required to make 12 assumptions to reduce the frame in to a statically determinate structure. From the deformed shape of the frame, it is observed that inflexion point (point of zero moment) occur at mid height of each column and mid point of each beam. This leads to 10 assumptions. Depending upon how the remaining two assumptions are made, we have two different methods of analysis: (i) Portal method and (ii) cantilever method. They are discussed in the subsequent sections.

(i) Portal method

In this method following assumptions are made.

1) An inflexion point occurs at the mid height of each column.
2) An inflexion point occurs at the mid point of each girder/beam.

3) The total horizontal shear at each storey is divided between the columns of that storey such that the interior column carries twice the shear of exterior column.

The last assumption is clear, if we assume that each bay is made up of a portal thus the interior column is composed of two columns (Figure 9.6). Thus the interior column carries twice the shear of exterior column. This method is illustrated in example 9.2.

Example 9.2

Analyse the frame shown in Figure 9.7a and evaluate approximately the column end moments, beam end moments and reactions.

Solution:

The problem is solved by equations of statics with the help of assumptions made in the portal method. In this method we have hinges/inflexion points at mid height of columns and beams. Taking the section through column hinges M .N , O we get, (ref. Figure 9.7b).

$$\sum F_X = 0 \qquad \Rightarrow \qquad V + 2V + V = 20$$

or $V = 5$ kN

Taking moment of all forces left of hinge R about R gives,

$$V \times 1.5 - M_y \times 2.5 = 0$$

$$M_y = 3\ \text{kN}(\downarrow)$$

Column and beam moments are calculated as,

$$M_{CB} = 5 \times 1.5 = 7.5\ \text{kN.m} \ ; \ M_{IH} = +7.5\ \text{kN.m}$$

$$M_{CF} = -7.5\ \text{kN.m}$$

Taking moment of all forces left of hinge S about S gives,

$$5 \times 1.5 - O_y \times 2.5 = 0$$

$$O_y = 3\text{kN}(\uparrow)$$

$$N_y = 0$$

Taking a section through column hinges J , K, L we get, (ref. Figure 9.7c).

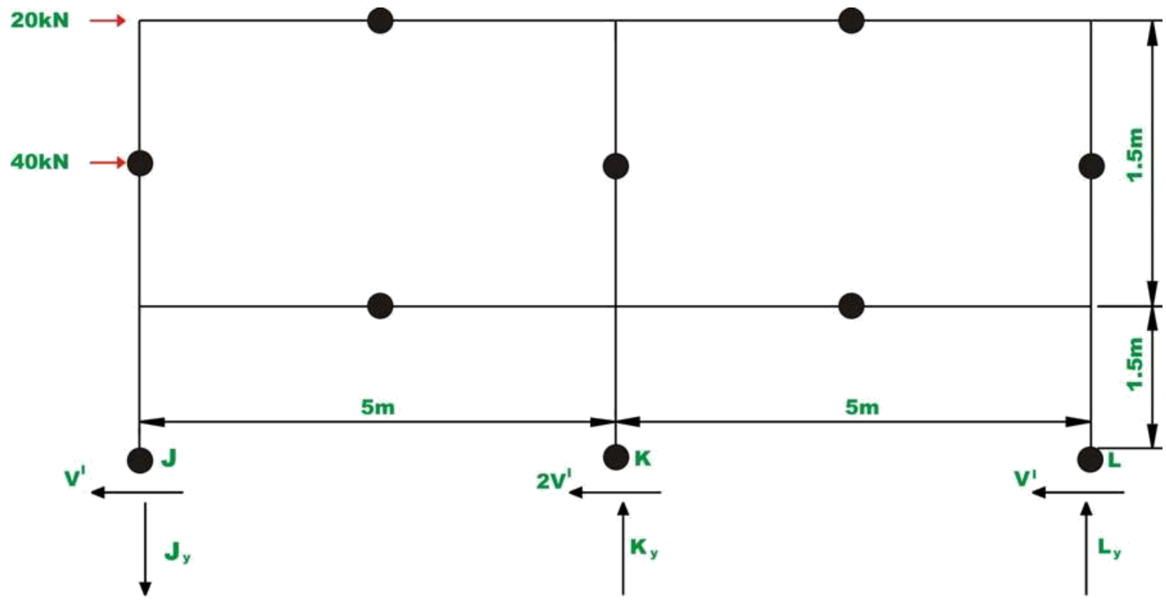


Figure 9.7c

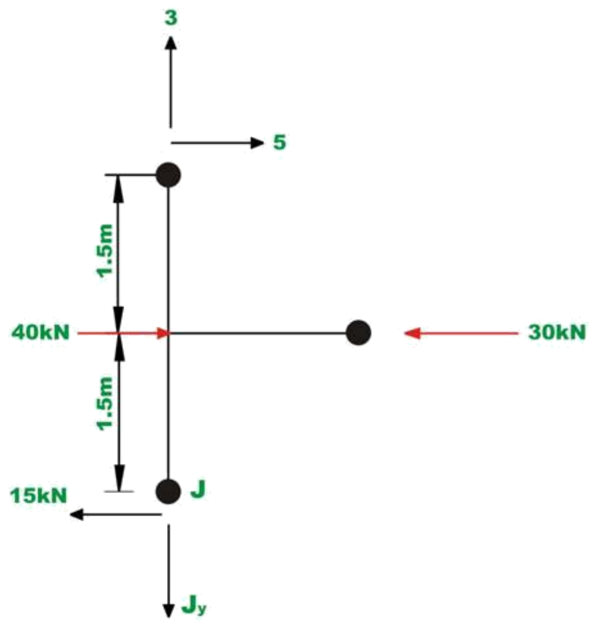


Figure 9.7d

$\sum F_X = 0 \quad \Rightarrow \quad V' + 2V' + V' = 60$

or $V' = 15$ kN.

Taking moment of all forces about P gives (vide Figure 9.7d)

$$\sum M_p = 0 \qquad 15 \times 1.5 + 5 \times 1.5 + 3 \times 2.5 - J_y \times 2.5 = 0$$

$$J_y = 15 \text{ kN} (\downarrow)$$

$$L_y = 15 \text{ kN} (\uparrow)$$

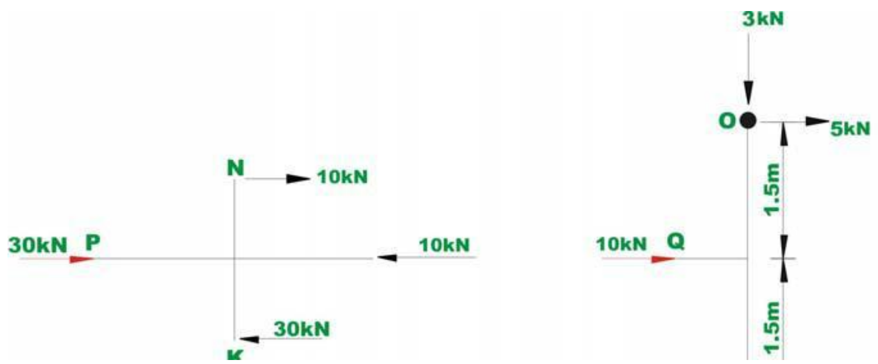


Figure 9.7e

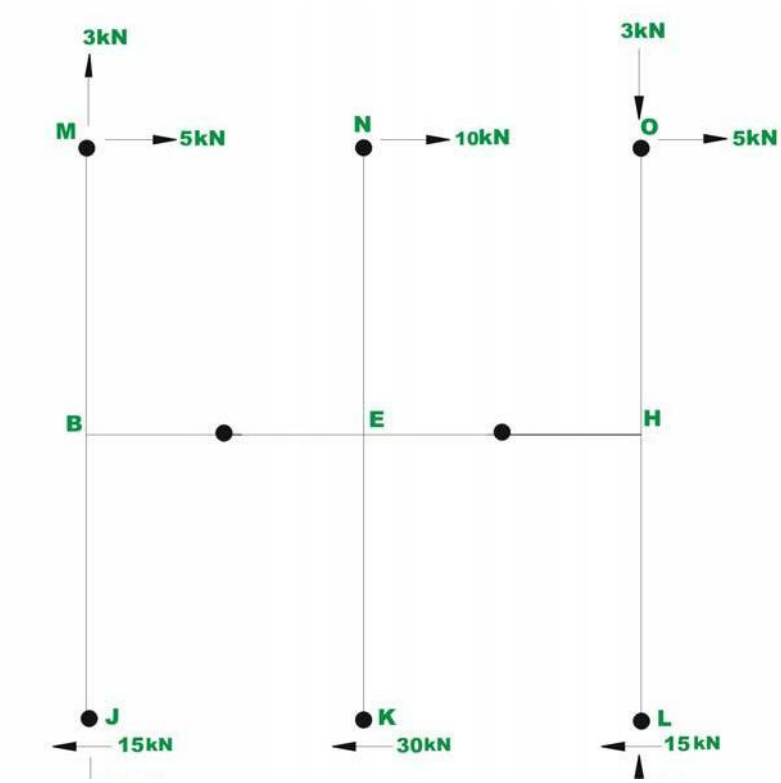


Figure 9.7f

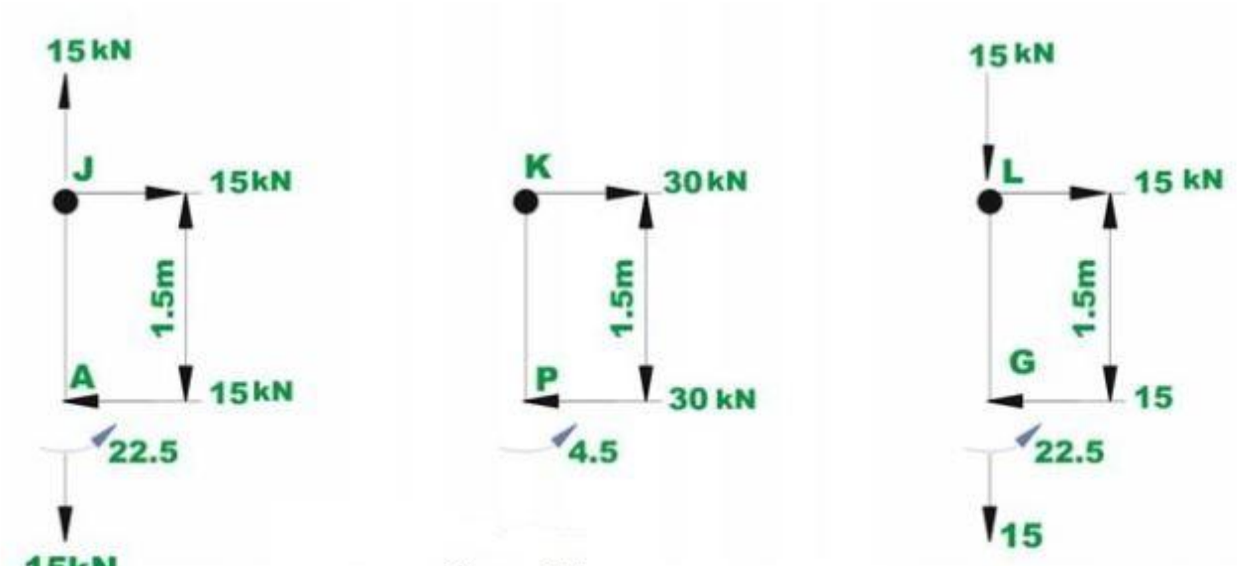


Figure 9.7g

Column and beam moments are calculated as, (ref. Figure 9.7f)

$$M_{BC} = 5\times1.5 = 7.5 \text{ kN.m} ; \; M_{BA} = 15\times1.5 = 22.5 \text{ kN.m}$$

$$M_{BE} = -30 \text{ kN.m}$$

$$M_{EF} = 10\times1.5 = 15 \text{ kN.m} ; M_{ED} = 30\times1.5 = 45 \text{ kN.m}$$

$$M_{EB} = -30 \text{ kN.m} \; ; \quad M_{EH} = -30 \text{ kN.m}$$

$$M_{HI} = 5\times1.5 = 7.5 \text{ kN.m} ; M_{HG} = 15\times1.5 = 22.5 \text{ kN.m}$$

$$M_{HE} = -30 \text{ kN.m}$$

Reactions at the base of the column are shown in Figure 9.7g**.**

Cantilever method

The cantilever method is suitable if the frame is tall and slender. In the cantilever method following assumptions are made.

1) An inflexion point occurs at the midpoint of each girder/beam.
2) An inflexion point occurs at mid height of each column.
3) In a storey, the intensity of axial stress in a column is proportional to its horizontal distance from the centre of gravity of all the columns in that storey. Consider a cantilever beam acted upon by a horizontal load P as shown in Figure 9.8. In such a column the bending stress in the column cross section varies linearly from its neutral axis. The last assumption in the cantilever method is based on this fact. The method is illustrated in example 9.3.

Example 9.3

Estimate approximate column reactions, beam and column moments using cantilever method of the frame shown in Figure 9.8a. The columns are assumed to have equal cross sectional areas.

Solution:

This problem is already solved by portal method. The centre of gravity of all column passes through centre column.

$$\bar{x} = \frac{\sum xA}{\sum A} = \frac{(0)A + 5A + 10A}{A + A + A} = 5\,\text{m} \quad \text{(from left column)}$$

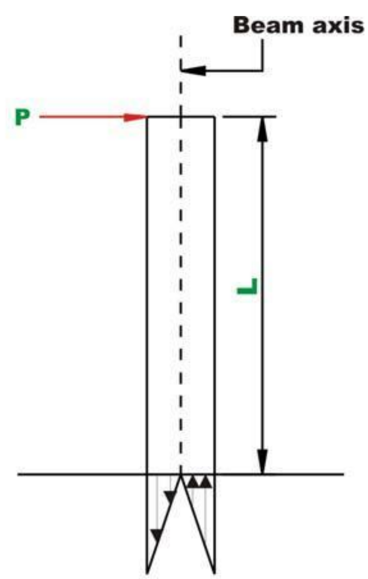


Figure 9.8a Cantilever Column

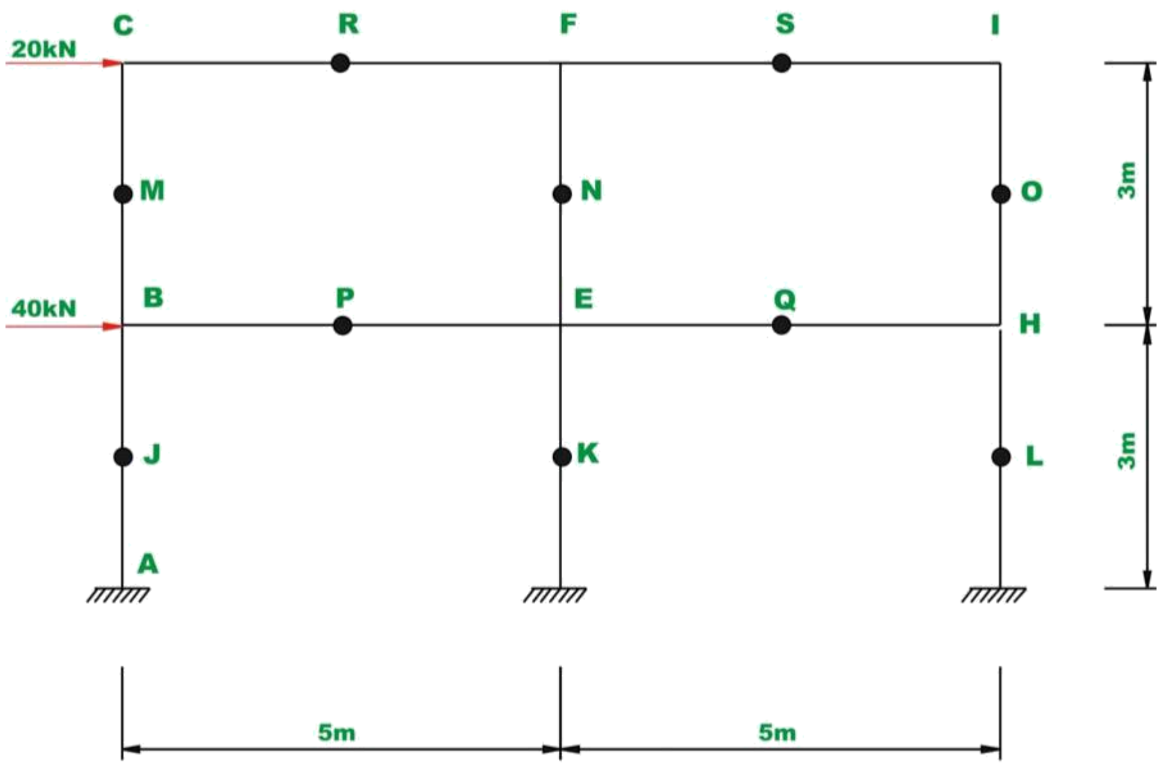


Figure 9.8b

Taking a section through first storey hinges gives us the free body diagram as shown in Figure 9.8b. Now the column left of C.G. *i.e;* CB must be subjected to tension and one on the right is subjected to compression.
From the third assumption,

$$\frac{M_y}{5\times A} = -\frac{O_y}{5\times A} \qquad \Rightarrow \qquad M_y = -O_y$$

Taking moment about O of all forces gives,

$$20\times 1.5 - M_y \times 10 = 0$$

$$M_y = 3\,\text{kN}(\downarrow) \quad ; \qquad O_y = 3\,\text{kN}(\uparrow)$$

Taking moment about R of all forces left of R ,

$$V_M \times 1.5 - 3 \times 2.5 = 0$$

$$V_M = 5 \ \text{kN}(\leftarrow)$$

Taking moment of all forces right of S about S ,

$$V_O \times 1.5 - 3 \times 2.5 = 0 \quad \Rightarrow \quad V_O = 5\,\text{kN.}$$

$$\sum F_X = 0 \qquad V_M + V_N + V_O - 20 = 0$$

$$V_N = 10\,\text{kN.}$$

Moments:

M_{CB} = 5 x 1.5 = 7.5 kN.m

M_{CF} = -7.5 kN.m

M_{FE} = 15 kN.m

M_{FC} = -7.5 kN.m

M_{FI} = -7.5 kN.m

M_{HI} = 7.5 kNm

M_{IF} = -7.5 kN.m

Take a section through hinges J , K, L (ref. Figure 9.8c). Since the centre of gravity passes through centre column the axial force in that column is zero.

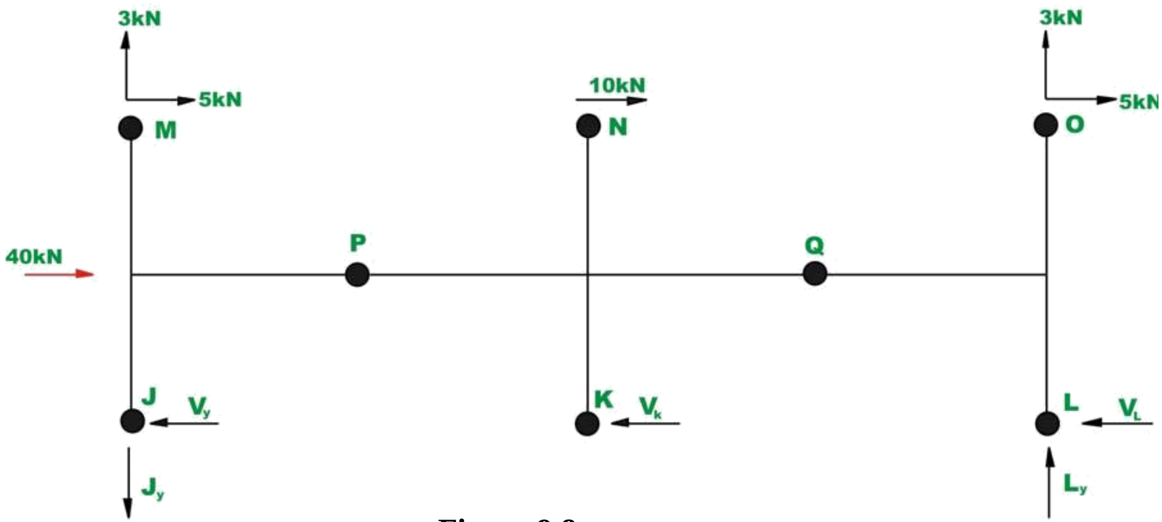


Figure 9.8c

Taking moment about hinge L , J_y can be evaluated. Thus,

$$20 \times 3 + 40 \times 1.5 + 3 \times 10 - J_y \times 10 = 0$$

$$J_y = 15\text{kN}(\downarrow) \quad ; \quad L_y = 15\text{kN}(\uparrow)$$

Taking moment of all forces left of P about P gives,

$$5 \times 1.5 + 3 \times 2.5 - 15 \times 2.5 + V_j \times 1.5 = 0$$

$$V_j = 15 \text{ kN}(\leftarrow)$$

Similarly taking moment of all forces right of Q about Q gives,

$$5 \times 1.5 + 3 \times 2.5 - 15 \times 2.5 + V_L \times 1.5 = 0$$

$$V_L = 15\text{kN}(\leftarrow)$$

$$\sum F_X = 0; \quad V_J + V_K + V_L - 60 = 0$$

$$V_K = 30 \text{ kN.}$$

Moments

$$M_{BC} = 5 \times 1.5 = 7.5 \text{ kN.m} \quad ; \quad M_{BA} = 15 \times 1.5 = 22.5 \text{ kN.m}$$

$$M_{BE} = -30 \text{ kN.m}$$

$$M_{EF} = 10 \times 1.5 = 15 \text{ kN.m} \quad ; \quad M_{ED} = 30 \times 1.5 = 45 \text{ kN.m}$$

$$M_{EB} = -30 \text{ kN.m} \qquad M_{EH} = -30 \text{ kN.m}$$

$$M_{HI} = 5 \times 1.5 = 7.5 \text{ kN.m} \quad ; \quad M_{HG} = 15 \times 1.5 = 22.5 \text{ kN.m}$$

$$M_{HE} = -30 \text{ kN.m}$$

Summary

In this lesson, the building frames are analysed by approximate methods. Towards this end, the given indeterminate building fame is reduced into a determinate structure by suitable assumptions. The analysis of building frames to vertical loads was discussed in section 9.2. In section 9.3, analysis of building frame to horizontal loads is discussed. Two different methods are used to analyse building frames subjected to horizontal loads: portal and cantilever method. Typical numerical problems are solved to illustrate the procedure.